Generalized magnetic tilt-Euler deconvolution

Ahmed Salem, Getech; Richard Smith, Fugro Airborne Surveys; Simon Williams, Getech; Dhananjay Ravat, Southern Illinois University Carbondale; and Derek Fairhead, Getech.*

Summary

We introduce Euler deconvolution of the magnetic tilt angle. The magnetic tilt angle is a generalized definition of the local phase. In the Generalized Tilt-Euler deconvolution method, no structural index is required. As a result the method can provide automatic estimates of source location from gridded magnetic data using second order derivatives. The method is tested using theoretical data for simple and complicated magnetic models and provided good results.

Introduction

One important goal in the interpretation of magnetic data is to determine the type and location of body that is the source of the magnetic anomaly. This has become particularly important recently because large volumes of magnetic data are being collected for environmental and geological applications. To realize this goal, a variety of semi-automatic methods, based on the use of derivatives of the magnetic field have been developed for the determination of magnetic source parameters such as locations of boundaries and depths (e.g., see references in Blakely, 1995; Nabighian et al., 2005). As faster computers and commercial software have become widely available, these techniques are being used more extensively.

Salem et al. (2005) presented ELW for interpreting profile magnetic data. They applied Euler deconvolution to the local phase instead of the magnetic field data. With this modification, it is possible to find automatic estimates of the source location regardless of the nature of the source type from profile magnetic data. In this paper, we extend the idea of the ELW method to gridded magnetic data (called here the Generalized Tilt-Euler deconvolution). We introduce Euler deconvolution of the magnetic tilt angle, where the magnetic tilt angle is a generalized definition of the local phase. In the Generalized Tilt-Euler deconvolution method, no structural index is required. As a result the method can provide automatic estimate of the source location and source type from gridded magnetic data. The method only requires second-order derivatives of the field and thus has potential advantages over methods requiring third order derivatives.

Tilt angle

The tilt angle was first proposed by Miller and Singh (1994) as a tool for locating magnetic sources on profile data. Verduzco et al. (2004) generalized the concept so that it could be applied to grid data by defining a generalized tilt angle as

$$\theta = \tan^{-1}\left[\frac{\partial M}{\partial z} \bigg/ \frac{\partial M}{\partial h}\right], \quad (1)$$

where

$$\frac{\partial M}{\partial h} = \sqrt{\left(\frac{\partial M}{\partial x}\right)^2 + \left(\frac{\partial M}{\partial y}\right)^2}, \quad (2)$$

and $\frac{\partial M}{\partial x}$, $\frac{\partial M}{\partial y}$ and $\frac{\partial M}{\partial z}$ are the derivatives of the magnetic field M in the x, y and z directions. The tilt angle has many interesting properties (Cooper and Cowan, 2006). For example, it responds equally to shallow and deep sources and can deal with a large dynamic range of amplitudes for sources at the same level. Also, due to the nature of the arctan trigonometric function, all tilt amplitudes are restricted to values between $-\pi/2$ and $\pi/2$ regardless of the amplitude of the vertical or the absolute value of the total horizontal gradient. This fact makes calculating the tilt angle similar to an automatic-gain-control filter: both operations tend to equalize the amplitude output of the magnetic anomalies across a grid or a profile (Verduzco et al., 2004). This can facilitate the interpretation of magnetic data where the anomalies cover a wide range of amplitudes. The tilt angle is identical to the local phase (Bracewell, 1965), which is used by Thurston and Smith (1997), in the SPI method for profile magnetic data. Our method extends the idea of the ELW method (Salem et al., 2005), in which Euler deconvolution is applied to the local phase.

The derivatives of the tilt angle k_x, and k_y have the same characteristics of the local wavenumber for 2D sources extending in the y and x directions respectively (Thurston and Smith (1997).

The Generalized magnetic tilt-Euler deconvolution

Similar to Salem et al. (2005), we define the rate of change of the generalized tilt angle θ with respect to the x, y, and z directions as

$$k_x = \frac{\partial \theta}{\partial x} = \frac{1}{|A|^2}\left(\frac{\partial M}{\partial h}\frac{\partial^2 M}{\partial x \partial z} - \frac{\partial M}{\partial z}\left(\frac{\partial M}{\partial h}\right)^{-1}\left(\frac{\partial M}{\partial x}\frac{\partial^2 M}{\partial x^2} + \frac{\partial M}{\partial y}\frac{\partial^2 M}{\partial y \partial x}\right)\right) \quad (3)$$

$$k_y = \frac{\partial \theta}{\partial y} = \frac{1}{|A|^2}\left(\frac{\partial M}{\partial h}\frac{\partial^2 M}{\partial y \partial z} - \frac{\partial M}{\partial z}\left(\frac{\partial M}{\partial h}\right)^{-1}\left(\frac{\partial M}{\partial x}\frac{\partial^2 M}{\partial x \partial y} + \frac{\partial M}{\partial y}\frac{\partial^2 M}{\partial y^2}\right)\right) \quad (4)$$

and

Generalized Tilt-Euler

$$k_z = \frac{\partial\theta}{\partial z} = \frac{1}{|A|^2}\left(\frac{\partial M}{\partial h}\frac{\partial^2 M}{\partial z^2} - \frac{\partial M}{\partial z}\left(\frac{\partial M}{\partial h}\right)^{-1}\left(\frac{\partial M}{\partial x}\frac{\partial^2 M}{\partial x\partial z} + \frac{\partial M}{\partial y}\frac{\partial^2 M}{\partial y\partial z}\right)\right) \quad (5)$$

where $A = \sqrt{\left(\frac{\partial M}{\partial x}\right)^2 + \left(\frac{\partial M}{\partial y}\right)^2 + \left(\frac{\partial M}{\partial z}\right)^2}$ is the total gradient of the magnetic field.

The 3D form of Euler's equation can be defined (Reid et al., 1990) as

$$(x - x_o)\frac{\partial M}{\partial x} + (y - y_o)\frac{\partial M}{\partial y} + (z - z_o)\frac{\partial M}{\partial z} = -\eta M \quad , \quad (6)$$

where x, y and z are the observation coordinates x_o, y_o, and z_o are the source coordinates, and η is a value that describes the anomaly attenuation rate that is known as the structural index. Following Salem et al. (2005) and taking the derivatives of the Euler equation (6) in the x, y and z directions, we obtain

$$(x - x_o)\frac{\partial^2 M}{\partial x^2} + (y - y_o)\frac{\partial^2 M}{\partial y\partial x} + (z - z_o)\frac{\partial^2 M}{\partial z\partial x} = -(\eta + 1)\frac{\partial M}{\partial x} \quad , \quad (7)$$

$$(x - x_o)\frac{\partial^2 M}{\partial x\partial y} + (y - y_o)\frac{\partial^2 M}{\partial y^2} + (z - z_o)\frac{\partial^2 M}{\partial z\partial y} = -(\eta + 1)\frac{\partial M}{\partial y} \quad , \quad (8)$$

and

$$(x - x_o)\frac{\partial^2 M}{\partial x\partial z} + (y - y_o)\frac{\partial^2 M}{\partial y\partial z} + (z - z_o)\frac{\partial^2 M}{\partial z^2} = -(\eta + 1)\frac{\partial M}{\partial z} \quad . \quad (9)$$

Multiplying equations (7) and (9) by $\left(\frac{1}{|A|^2}\frac{\partial M}{\partial z}\frac{\partial M}{\partial x}\right)$ and $\left(\frac{1}{|A|^2}\left(\frac{\partial M}{\partial x}\right)^2\right)$ respectively, and subtracting the first from the second, we obtain

$$\begin{aligned}
&\frac{(x - x_0)}{|A|^2}\left(\frac{\partial^2 M}{\partial x\partial z}\left(\frac{\partial M}{\partial x}\right)^2 - \frac{\partial^2 M}{\partial x^2}\frac{\partial M}{\partial x}\frac{\partial M}{\partial z}\right) + \\
&\frac{(y - y_0)}{|A|^2}\left(\frac{\partial^2 M}{\partial y\partial z}\left(\frac{\partial M}{\partial x}\right)^2 - \frac{\partial^2 M}{\partial y\partial x}\frac{\partial M}{\partial x}\frac{\partial M}{\partial z}\right) + \\
&\frac{(z - z_0)}{|A|^2}\left(\frac{\partial^2 M}{\partial z^2}\left(\frac{\partial M}{\partial x}\right)^2 - \frac{\partial^2 M}{\partial x\partial z}\frac{\partial M}{\partial x}\frac{\partial M}{\partial z}\right) = 0
\end{aligned} \quad . \quad (10)$$

Multiplying equations (8) and (9) by $\left(\frac{1}{|A|^2}\frac{\partial M}{\partial z}\frac{\partial M}{\partial y}\right)$ and $\left(\frac{1}{|A|^2}\left(\frac{\partial M}{\partial y}\right)^2\right)$ respectively, followed by their subtraction, we obtain

$$\begin{aligned}
&\frac{(x - x_0)}{|A|^2}\left(\frac{\partial^2 M}{\partial x\partial z}\left(\frac{\partial M}{\partial y}\right)^2 - \frac{\partial^2 M}{\partial x\partial y}\frac{\partial M}{\partial y}\frac{\partial M}{\partial z}\right) + \\
&\frac{(y - y_0)}{|A|^2}\left(\frac{\partial^2 M}{\partial y\partial z}\left(\frac{\partial M}{\partial y}\right)^2 - \frac{\partial^2 M}{\partial y^2}\frac{\partial M}{\partial y}\frac{\partial M}{\partial z}\right) + \\
&\frac{(z - z_0)}{|A|^2}\left(\frac{\partial^2 M}{\partial z^2}\left(\frac{\partial M}{\partial y}\right)^2 - \frac{\partial^2 M}{\partial y\partial z}\frac{\partial M}{\partial y}\frac{\partial M}{\partial z}\right) = 0
\end{aligned} \quad (11)$$

Adding (10) and (11) we obtain

$$\begin{aligned}
&\frac{(x - x_0)}{|A|^2}\left(\left(\frac{\partial M}{\partial h}\right)^2\frac{\partial^2 M}{\partial x\partial z} - \frac{\partial M}{\partial z}\left(\frac{\partial M}{\partial x}\frac{\partial^2 M}{\partial x^2} + \frac{\partial M}{\partial y}\frac{\partial^2 M}{\partial x\partial y}\right)\right) + \\
&\frac{(y - y_0)}{|A|^2}\left(\left(\frac{\partial M}{\partial h}\right)^2\frac{\partial^2 M}{\partial y\partial z} - \frac{\partial M}{\partial z}\left(\frac{\partial M}{\partial x}\frac{\partial^2 M}{\partial x\partial y} + \frac{\partial M}{\partial y}\frac{\partial^2 M}{\partial y^2}\right)\right) + \\
&\frac{(z - z_0)}{|A|^2}\left(\left(\frac{\partial M}{\partial h}\right)^2\frac{\partial^2 M}{\partial z^2} - \frac{\partial M}{\partial z}\left(\frac{\partial M}{\partial x}\frac{\partial^2 M}{\partial x\partial z} + \frac{\partial M}{\partial y}\frac{\partial^2 M}{\partial z\partial y}\right)\right) = 0
\end{aligned} \quad (12)$$

Dividing (12) by $\frac{\partial M}{\partial h}$ and substituting with the definitions of the tilt angle derivatives (equations 3, 4, and 5), we obtain

$$k_x x + k_y y + k_z z = k_x x_0 + k_y y_0 + k_z z_0 \, . \quad (13)$$

Equation (13) is the Euler deconvolution of the magnetic tilt angle. It is similar to the conventional Euler deconvolution but applied to the derivatives of the magnetic tilt angle.

Theoretical tests

In this section, we show results of testing the method using theoretical data for simple and complicated magnetic models. Figure 1a shows the synthetic magnetic anomaly for a prism model with magnetic inclination of 60°, declination 0°, and a depth to the top of 3 km. The magnetic data were calculated at an interval of 1 km. Depth and structural index estimates were generated using a window size of 11 x 11 grid points.

The peak amplitudes of the tilt derivatives are displayed in Figures 1b to 1d. The derivatives in the x and y directions exhibit asymmetric peaks. The vertical derivative of the tilt varies rapidly over the prism edge – the edge itself corresponds either to a sharp peak or trough, flanked by two troughs or two peaks respectively.

Figures 1e and 1f show in a 3D view the estimates of source location and structural index respectively, derived from Euler deconvolution of the tilt angle. The source depths have a mean value of 2.951 km with standard deviation 0.067 km. The estimated depths are shallower on the southeast and southwest edges, with a mean depth for these source estimates of 2.896 km compared to a mean value of 2.992 km on the northeast and northwest edges. The estimated values of structural index are slightly greater on the northeast and northwest edges, with an overall mean value of -0.005 and standard deviation (0.0406).

The Bishop model has been used by various authors to test methods of estimating source depths from magnetic data (Williams et al. 2002, 2005; Fairhead et al., 2004; and Reid et al., 2005) and has been the subject of a workshop at the 2006 SEG annual meeting. The basement is assumed to extend down to 20 km depth and be overlain by nonmagnetic sediments. Figure 2a shows the magnetic response for a basement susceptibility varying from 1×10^{-3} to 8×10^{-3} cgs units and an ambient geomagnetic field with strength 50 000 nT, inclination 90°, and declination 0°.

Figures 2b to 2d show the derivatives of the tilt angle. Peaks and troughs in the grids of k_x and k_y (Figures 2b-c) help to define structures oriented in the x and y directions respectively. The edges of features in the model such as strong magnetization contrasts correspond to zero-crossings in the grid of k_z (Figure 2d), though there are many additional zero-crossings that do not have an obvious correlation to features in the model basement.

Figure 2f and 2g show the estimates of the source location and structural index based on Euler deconvolution of the tilt angle. These results are derived using a window of 11 x 11 grid points (5 km x 5 km). Poor solutions are rejecting using the following strategy – solutions are accepted if the x,y location lies within 2 km of peaks in the grid of k_h, if the estimated depth lies between 0 km and 12 km, and if the estimated structural index is between –0.5 and 1.5. The overall pattern of estimated depths (Figure 2e) correlates reasonably well with the model basement depth. The estimates of structural index (Figure 2f) are consistently in the range 0-0.5 over the two linear magnetization contrasts in the basement, but show considerable scatter for other sources.

The vertical scatter in the depth estimates is difficult to determine from Figure 2f alone, so we illustrate the degree of correlation between the depth and structural index estimates and model basement further with a cross-section. Figure 3a shows a cross-section through the Bishop model perpendicular to the dominant trend of topographic features in the model basement.

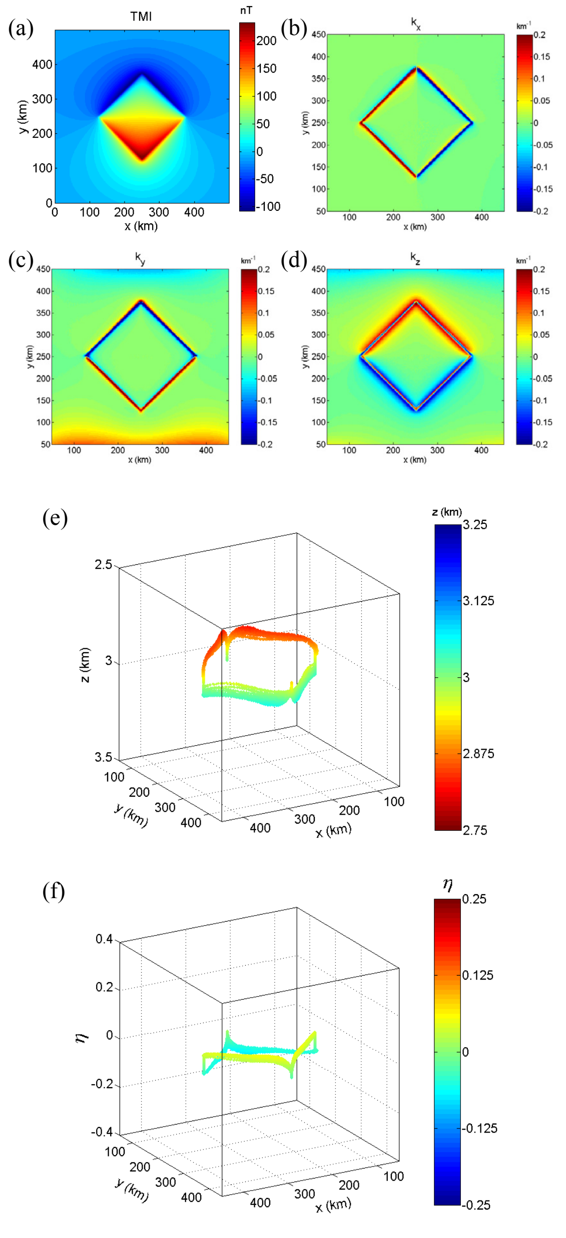

Figure 1: (a) TMI anomaly map for prism model described in text, (b-d) derivatives of the tilt angle in the x, y and z directions, (e) estimates of source location and (f) estimates of structural index from Euler deconvolution of the tilt angle.

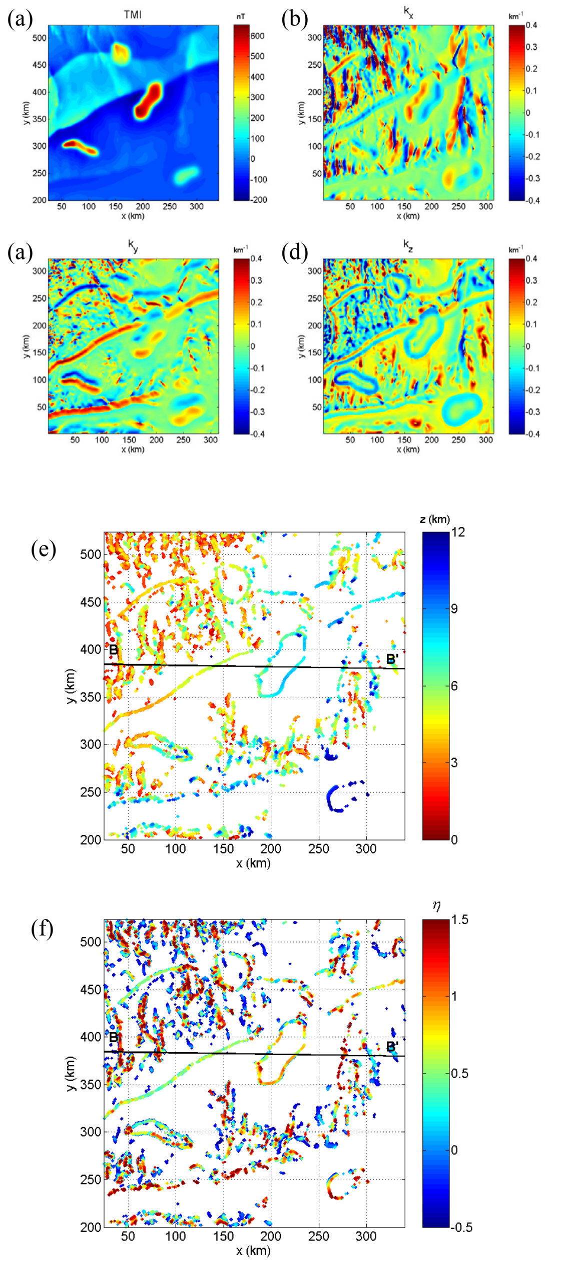

Figure 2: (a) TMI anomaly map for Bishop model, (b-d) derivatives of the tilt angle in the *x*, *y* and *z* directions, (e) estimates of source location and (f) estimates of structural index from Euler deconvolution of the tilt angle.

For this cross-section, the points plotted are all source locations within a band extending 4 km either side of the profile, projected onto the profile perpendicular to the profile direction. The depth estimates (Figure 3b) at *x* less than 100 km (where the basement is relatively shallow) exhibit several kilometers of vertical scatter. For depth estimates at *x* greater than 100 km, the source locations form tight clusters which correlate well with the model basement. The structural index estimates (Figures 3c) for any individual source show a considerable amount of scatter. For non-idealized sources, we would expect the structural index to vary with the position of the data window relative to the source (Ravat, 1996), so it is not surprising that different window locations give different structural index estimates for any given source.

Conclusion

We present Euler deconvolution of the magnetic tilt angle, which is a generalized definition of the local phase. The method requires no structural index and can provide automatic estimates of the source location from gridded magnetic data. The method is tested using theoretical data for a single prism model and Bishop model. In both cases, it provided good results. As with many semi-automatic method for magnetic depth estimation, the next challenge is to develop criteria to identify the robust solutions and reject those that are poorly constrained.

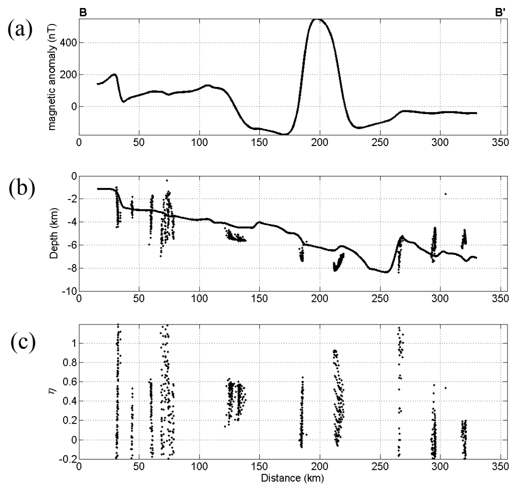

Figure 3: Cross section through results shown in figure 2e-f. (a) TMI anomaly, (b) Estimates of source location (solid line shows model basement depth), (c) Estimates of structural index.