Tilt-depth method: A simple depth estimation method using first-order magnetic derivatives

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Aeromagnetic data are routinely presented as contour or color-shaded maps of the total magnetic intensity (TMI). An interpreter's task is to identify features (anomalies) contained within the map and qualitatively and/or quantitatively interpret them into geologic structures at depth. If the map contains anomalies that have large magnetic intensities, the bodies might be considered to have large magnetizations, or to be at shallow depths. Small amplitude anomalies superimposed on these anomalies could be masked or even missed by an interpreter. Thus the task of the interpreter is to use the spectral content of the anomalies to try and resolve these ambiguities. Part of this process is also to obtain estimates of the depth and shape of the body causing the anomalies.

An interpretation difficulty with TMI anomalies is that they are dipolar (anomalies having positive and negative components) such that the shape and phase of the anomaly depends in part on the magnetic inclination and the presence of any remanent magnetization. This anomaly complexity makes interpretation more difficult because the body and its edges do not necessarily coincide with the most obvious mapped feature (e.g., anomaly maxima). The reduction-to-the-pole (RTP) technique transforms TMI anomalies to anomalies that would be measured if the field were vertical (assuming there is only an inducing field). This RTP transformation makes the shape of magnetic anomalies more closely related to the spatial location of the source structure and makes the magnetic anomaly easier to interpret, as anomaly maxima will be located centrally over the body (provided there is no remanent magnetization present).

To map the edges of bodies, the horizontal derivatives of the RTP field or of the pseudogravity field are often used. In both cases, the horizontal derivative will peak above a vertical contact. However, a dipping contact, an incorrect inclination used in RTP transformation or the presence of remanent magnetization, will tend to shift anomaly maxima away from the true location of the contact. In general, the interpreter's ability to avoid these complexities in a simple manner can have immense advantages. In this paper, we present a simple method of estimating the depth of magnetic source bodies (assuming a vertical-contact model) from just contours of the magnetic tilt angle map. The magnetic tilt angle is a normalized derivative based on the ratio of the vertical and horizontal derivatives of the RTP field. We call this new method the "tilt-depth method" which provides an intuitive means of understanding the variation in depth of magnetic source bodies (or magnetic basement as shown with the field example). Its main advantage is it can be used by nonspecialists and is independent of any need for more advanced numerical analysis of the data. The method in its simplest form assumes the source structures have vertical contacts, there is no remanent magnetization, and the magnetization is vertical.

Method. The tilt angle was first described by Miller and Singh (1994), before being further refined by Verduzco (2004) and GETECH and defined as

$$\theta = \tan^{-1}\left[\frac{\partial M}{\partial z}\bigg/\frac{\partial M}{\partial h}\right], \qquad (1)$$

where

$$\frac{\partial M}{\partial h} = \sqrt{\left(\frac{\partial M}{\partial x}\right)^2 + \left(\frac{\partial M}{\partial y}\right)^2},$$

and

$$\frac{\partial M}{\partial x}, \frac{\partial M}{\partial y}\ \frac{\partial M}{\partial z}$$

are first-order derivatives of the magnetic field M in the x, y, and z directions. The tilt angle has many interesting properties. For example, due to the nature of the arctan trigonometric function, all tilt amplitudes are restricted to values between –90° and +90° regardless of the amplitude of the vertical or the absolute value of the total horizontal gradient. This fact makes calculating the tilt angle similar but superior to an AGC filter in that, besides equalizing the amplitude output of the magnetic anomalies across a grid or along a profile, it retains spectral integrity of the signal, allowing further quantitative analysis (e.g., determining local wavenumber).

The general expressions published by Nabighian (1972) for the vertical and horizontal derivatives of the magnetic field over contacts located at a horizontal location of h=0 and at a depth of z_c are

$$\frac{\partial M}{\partial h} = 2KFc\sin d\,\frac{z_c\cos(2I-d-90)+h\sin(2I-d-90)}{h^2+z_c^2}, \qquad (2)$$

$$\frac{\partial M}{\partial z} = 2KFc\sin d\,\frac{h\cos(2I-d-90)-z_c\sin(2I-d-90)}{h^2+z_c^2}, \qquad (3)$$

where K is the susceptibility contrast at the contact, F the magnitude of the magnetic field, c=1–cos^2 $i$sin^2$\boldsymbol{A}$, $\boldsymbol{A}$ the angle between the positive h-axis and magnetic north, i the ambient field inclination, tan$I$=tan$i$/cos$A$, d the dip (measured from the positive h-axis), and all trigonometric quantities are in degrees. Under certain assumptions such as when the contacts are nearly vertical and the magnetic field is vertical or RTP, Equations 2 and 3 can be written as

$$\frac{\partial M}{\partial h} = 2KFc\,\frac{z_c}{h^2+z_c^2} \qquad (4)$$

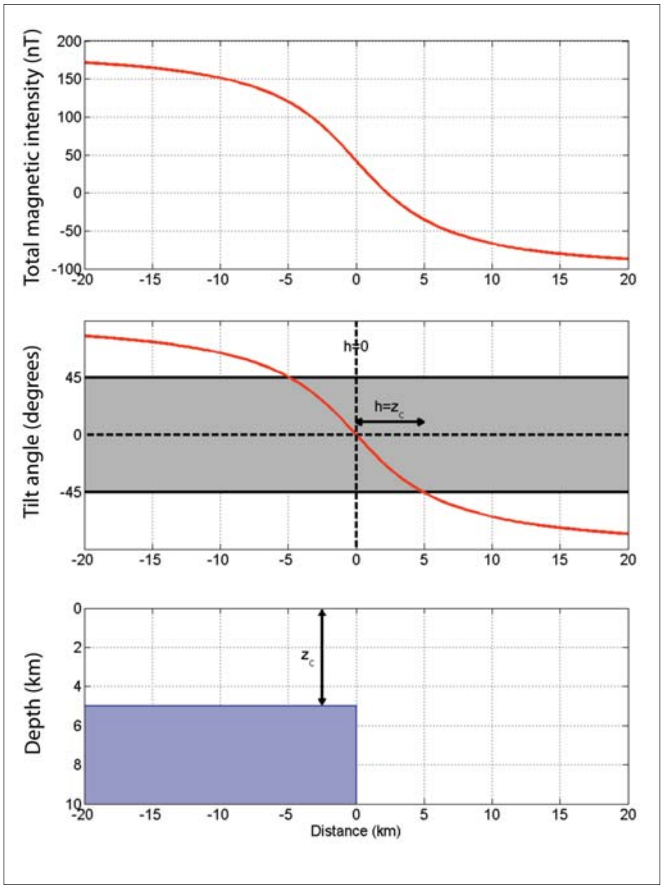

Figure 1. Profile model of (top) the magnetic anomaly, and (middle) the tilt derivative over a vertical contact (bottom) for RTP (or vertical inducing field). Tilt values are restricted to within ±90°. The contact coincides with the zero crossing and the part of the tilt derivative between ±45° is highlighted.

$$\frac{\partial M}{\partial z} = 2KFc\frac{h}{h^2 + z_c^2} \tag{5}$$

Substituting Equations 4 and 5 in 1, we get

$$\theta = \tan^{-1}\left[\frac{h}{z_c}\right] \tag{6}$$

Equation 6 indicates the value of the tilt angle above the edges of the contact is 0° (h=0) and equal to 45° when $h=z_c$ and –45° when $h=-z_c$. This suggests that contours of the magnetic tilt angle can identify both the location (θ=0°) and depth (half the physical distance between ±45° contours) of contact-like structures.

Theoretical examples. The profiles in Figure 1 demonstrate the relationship between the tilt angle and source depth for a vertical contact model. The anomaly is calculated as a north-south profile across an east-west striking source, for a magnetic inclination of 90°. The profile of the tilt angle passes through zero directly over the contact edge (h=0), and passes through the dashed lines marking ±45° at a distance from the edge equal to the source depth ($h=Z_c$). Note that our method is valid only for data that has been reduced-to-the-pole. Figure 1 of Verduzco et al. (2004) clearly demonstrates the asymmetry of profiles of the tilt angle for other magnetic inclinations.

Figure 2a shows the synthetic magnetic anomaly contour map for a model containing two vertical-sided prisms with the edge locations indicated by the dashed lines. The top of prism A is located at a depth of 4 km and prism B at a depth of 16 km. Both prisms are defined with effectively infinite depth extent, and with a positive magnetization contrast of 10^{-4} A/m. The inducing field has an inclination of 90°. The anomaly field is calculated on a regular grid with a spacing of 0.5 km. Figure 2b shows the magnetic tilt angle map generated from the data shown in Figure 2a. The region enclosed by the 45° and –45° contours is gray and the zero contour is shown by the dashed line (indicates an approximate location of the source edges). While the distance between the two ±45° contours and the 0° contours is not everywhere identical around the perimeter of each body, because of the anomaly interference and the breakdown of the two dimensionality assumption, we observe that the source depth is

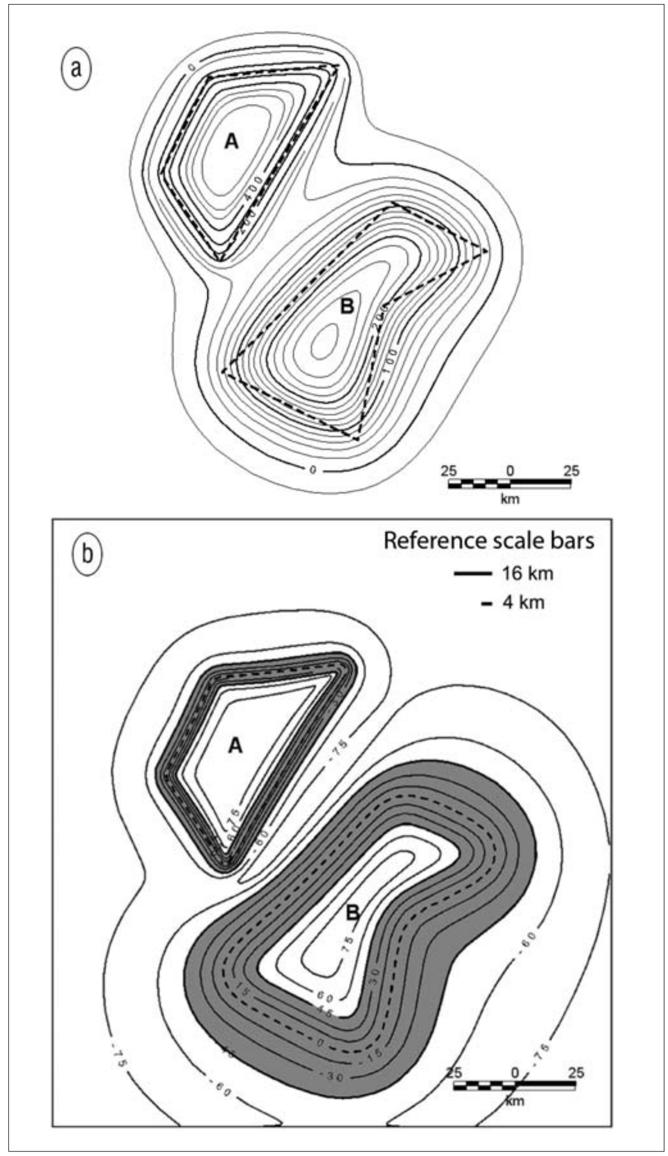

Figure 2. Synthetic magnetic test model. (a) The magnetic response of the synthetic test model containing two vertical-sided prisms with magnetizations of 10^{-4} A/m, and with the edge locations indicated by the dashed lines. The top of the upper left prism (A) is located at a depth of 4 km, the top of the lower right prism (B) at a depth of 16 km. The inducing field has an inclination of 90° and a declination of 0°. (b) Magnetic tilt angle map generated from (a). Dashed lines show the 0° contour of the tilt angle. Solid lines are contours of the tilt angle for –45° and 45°.

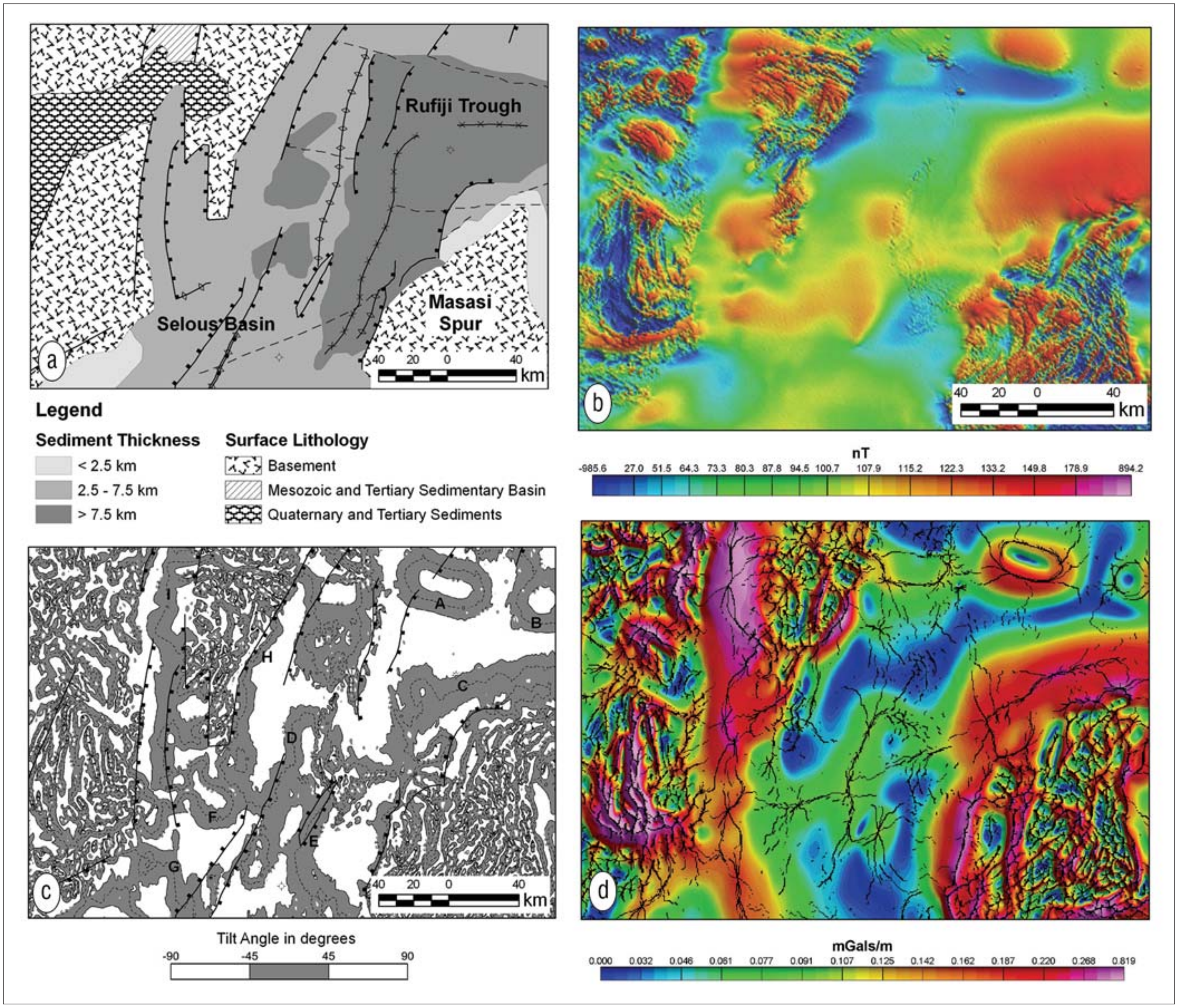

Figure 3. Field test data set. (a) Regional geologic structure of the Karoo Basin of southeast Tanzania. Adapted from Tanzania Petroleum Development Council promotion brochure (http://www.tpdc-tz.com). (b) TMI anomaly map for southeast Tanzania upward continued to 1 km. (c) Tilt-depth map showing contours of the tilt derivative for the Tanzania magnetic anomaly data after RTP and upward continued to a height of 1 km before the tilt was calculated. Dashed lines show the 0º contour of the tilt angle. Solid lines are contours of the tilt angle for –45º and 45º—the distance between these contours is approximately equal to twice the depth to the magnetic source assuming a contact-type source geometry. Some examples of depths (below terrain) indicated by the tilt contours within the Selous Basin and Rufiji Trough are labeled: A = 5 km, B = 5 km, C = 7-8 km, D = 4 km, E = 5–6 km, F = 3–4 km, G = 3–3.5 km, H = 1–2 km, I = 5–6 km. (d) Lineament analysis of the aeromagnetic field data using automated tracking of the maxima of the horizontal derivative of the pseudo gravity field.

roughly equivalent to half the width of the shaded strip delineating its edge (i.e., depth to the top of these sources models). The 2D imaging of the ±45° strips allows a spatial indication of where anomalies are suffering from interference as well as a rapid means of estimating the depth to the top edges of the sources in locations least affected by anomaly interference.

Field example. In this section, we demonstrate the tilt-depth method on aeromagnetic data over the Karoo sedimentary rift structures of southeast Tanzania. The regional geologic setting is a consequence of the breakup of Gondwana and rifting along the eastern margin of Africa. A schematic map of the geologic structure over the rift is shown in Figure 3a. The Selous Basin is a N–NE-trending rift basin infilled with up to ~10 km of mainly nonmarine and nonmagnetic sediments ranging in age from Permian-Triassic to Tertiary. The basin is bounded to the west and east by shallow basement with the Masasi Spur separating the Selous Basin from the coastal basins of eastern Tanzania. The Rufiji Trough is located to the NE of the Selous Basin and exhibits east-west extensional structures of Jurassic age superimposed on earlier N–NE trending structures.

A countrywide aeromagnetic grid for Tanzania has been compiled by GETECH from 1-km-spaced flight-line data oriented predominantly east-west, with mean terrain clearance of 120 m. The resulting grid has a nodal separation of 0.25 km. Figure 3b shows the TMI anomaly map over the study area and clearly delineates the rift basin outline by the shape change in the frequency content of the magnetic anomalies. Before applying the tilt-depth method, the data were converted to RTP using a magnetic inclination of –40º and a declination of –3.5º, and upward continued to a distance of 1 km. The upward continuation was found to be necessary to obtain the cleanest image of the structures based on the contours of the magnetic tilt angle. Figure 3c shows the simple form of the magnetic tilt angle map only displaying the contours of –45°, 0°, and 45°, and the areas bounded by these contours are shaded in grey. The tilt angle map over the shallow basement areas are characterized by high frequency magnetic anomalies and closely spaced contours. In contrast over the deep parts of the basin widely spaced contours are observed. Using the tilt-depth contour methodology, we can determine the width of these grey zones to provide an immediate estimate of the depth to basement and how it varies across the area.

The regions of shallow basement (the Masasi Spur, and the region west of the Selous Basin) are characterized by numerous lineaments in the tilt derivative, with the distance between the –45° and 45° contours typically less than 4 km. This distance is twice the source depth (assuming a contact source geometry), so after correcting for the continuation distance the approximate depths to magnetic sources indicated by the tilt contours are predominantly very shallow - no greater than 1 km beneath the surface. Since the flight line spacing is 1 km, aliasing of the anomalies could be a reason for the depths not being closer to the surface. Within the Selous Basin and Rufiji Trough, the tilt contours are much more widely spaced. These contours define magnetic lineaments within the basin, as well as areas of more chaotic contours which in part could be due to anomaly interference. Since the methodology assumes 2D vertical contacts, the basement depths in the Selous Basin (locations D to I) range from 3 to 6 km while in the Rufiji basin (locations A to C) they increase up to 8 km. These depths show a good correspondence with the regional variation in sediment thicknesses based on seismic and well control data as indicated by Figure 3a.

The advantage of the tilt-depth method is, however, its ability to identify those parts of anomaly structures that are least affected by interference where repeated depth estimates are most likely to be reliable. For completeness Figure 3d provides the plot of the horizontal derivative of the pseudo gravity with automatic tracking of their maxima using the Blakely and Simpson method to define the location of contacts. These contacts are also closely mapped by the dashed (0° tilt angle) contours in Figure 3c less the "spider legs" seen in Figure 3d that define small scale local 2D ridges within the grid. The results of Figures 3c and d should be viewed as stage products from which structural and depth maps can be constructed.

Discussion and conclusions. We present a simple and fast method to locate vertical contacts from RTP magnetic data. The tilt-depth method only depends on mapping specific contours of the magnetic tilt angles. The zero contours delineate the spatial location of the magnetic source edges whilst the depth to the source is the distance between the zero and either the –45° or the +45° contour or their average.

The tilt-depth method adds to the arsenal of geophysical methods currently in use to estimate magnetic source depths, many of which use second- and/or third-order derivatives. These include methods based on Euler's equation and the local wavenumber, both of which calculate the source depths for a range of source-body geometries, and, more recently, for the simultaneous estimation of both source depth and source type. The tilt-depth method by comparison can be considered to be both simple and elegant to derive. The two principal advantages of the method are its simplicity both in its theoretical derivation and in its practice application, and it provides both a qualitative and quantitative approach to interpretation by allowing the interpreter to visually inspect (spatially analyse) the tilt-depth map to identify locations where depth estimates may be compromised by interfering magnetic anomalies and locations where more reliable depth estimates can be made. These reliable locations can then be re-evaluated using different magnetic depth estimation methods.

Other advantages of the method are that by virtue of using first order derivatives, it is potentially less sensitive to noise in the data compared to methods relying on higher order derivatives, and unlike the Euler method there is no need to choose window size, nor is there a problem of solution clusters to contend with. The visual inspection advantage is clearly demonstrated in the field example using vintage digital aeromagnetic data, which contains noise sources that can be considerably reduced by modern high resolution surveys using superior magnetometers, acquisition methods, and GPS controlled navigation.

We believe there is ample scope to improve the method further and by making it less dependent on the need to process the TMI data to RTP, a particular problem close to the magnetic Equator, and having to assume a single source type structure. What we have presented here is hopefully a new, simpler way of qualitatively and quantitatively evaluating magnetic survey data that can be more readily appreciated by non-specialists.

Suggested reading. *Potential Theory in Gravity and Magnetic Applications* by Blakely (Cambridge, 1995). "Numerical calculation of the formula of reduction to the magnetic pole" by Baranov and Naudy (GEOPHYSICS, 1964). "Approximating edges of source bodies from magnetic or gravity anomalies" by Blakely and Simpson (GEOPHYSICS, 1986). "Enhancing potential field data using filters based on the local phase" by Cooper et al. (*Computers and Geosciences*, 2006). "Mapping basement magnetization zones from aeromagnetic data in the San Juan Basin, New Mexico" by Cordell and Grauch (*Utility of Regional Gravity and Magnetic Maps*, SEG, 1985). "The sedimentary basins of Tanzania—Reviewed" by Mbede (*Journal of African Earth Sciences*, 1991). "Potential field tilt—A new concept for location of potential field sources" by Miller and Singh (*Journal of Applied Geophysics*, 1994). "The analytic signal of two-dimensional magnetic bodies with polygonal cross-section; its properties and use for automated anomaly interpretation" by Nabighian (GEOPHYSICS, 1972). "The historical development of the magnetic method in exploration" by Nabighian et al. (GEOPHYSICS, 2005). "New insights into magnetic derivatives for structural mapping" by Verduzco et al. (*TLE*, 2004). TLE

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